Performance analysis of three unit redundant system with switch and human failure using Copula distribution

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Abstract. The performance of a three unit redundant system with the impact of switch and human failure has been studied. A system with the three identical units has been considered for assessment of performance under 2 out of 3: G; policy. In the system, a switch is used to transfer load from one unit to another unit. All three units of the system are connected in parallel configuration and working under 2-out-of-3: G; policy. The system can have two types of failure i.e. partial failure and complete failure. Partial failure degrades the efficiency of system; and the complete failure breakdown the system and stops its functioning. Switch failure and human failure are considered as complete failure. The system has two types of failure and two types of repair. General repair is employed to the partially failed system and Gumbel - Hougaard family copula distribution complete failed system. The system is studied by supplementary variable technique and various measures of reliability, such as availability, reliability, MTTF and profit functions have been discussed. Some particular cases have been discussed by taking different failure rates.

1 Introduction

References to the current trend of technological advancement, industrial systems are becoming smaller and more complex due to automation and miniaturization. In safety and critical applications, it becomes necessary to improve the reliability through $k$-out-of-$n$ redundancy. In contrast to improve reliability of the system, many researchers including \cite{1, 2, 3, 4} have highlighted and proposed their work, contributions by considering different types of mathematical models. Therefore, tracing and repair of fault units or components is sometimes becomes time consuming and difficult as well. It is required to have an idea of system configuration before design a new system. The architecture of system should be designed so that it must consistof some redundant unit in standby mode that might

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\end{itemize}
perform the intended task, whenever it is needed. Therefore, it is important to make a view of the system configuration before designing. In series configuration, the system fails when any one unit fails; and in parallel configuration, the system works with less efficiency until one unit of its configuration is in good condition. Both configurations are independent by nature and used to discuss reliability characteristics of a simply designed model. In the first, the chance of system failure is very high, but in the second, it will work with less efficiency. Therefore, it becomes necessary to study a k-out-of-n system in which system work successfully until k of its units is in good condition. Further, the k-out-of-n system configuration is categorized in k-out-of-n: G and k-out-of-n: F system. The redundant system configuration of the form: k-out-of-n system has wide application in industrial systems. The k-out-of-n system works, if and only if at least k of the n components works. Redundancy is a technique, which improve reliability and availability of system over the time. The k-out-of-n system plays a vital role in the process industrial and design and which have received attention of researchers. Many researchers including [5, 6, 7] have extensively studied redundant systems, and their contributions highlighted with its degree of completeness. The standby complex system have been extensively studied by various researchers in the reference to the study of standby complex systems Singh et al [6] discussed availability of standby complex system under waiting repair and human failure using Gumbel-Hougaard family copula distribution. Ram et al [7] studied stochastic analysis of a standby system with waiting repair strategy. The study clearly explains the importance of waiting time to repair and human error which seem to be possible in many engineering systems. Ram & Kumar [8] studied performance of a structure consisting a 2-out-of-3: F substructure under human failure using analytical approach to compute the reliability measures of a system. Ibrahim and Hussaini [9] focused on the comparative analysis of three unit redundant systems with three types of failure. The result obtained shows that preventive maintenance is better than other systems without preventive maintenance. Singh et al [10] studied a multi-state k-out-of-n type of system. The results for illustration have been highlighted specially for 2-out-of-3: G system. In continuation to the study of the repairable complex systems Ram & Kumar [11] studied performance analysis of a system under 1-out-of-2: G scheme with perfect reworking system. Singh et al [12] studied availability, MTTF and cost analysis of a system having two units in series configuration with controller and concluded that availability of the system decreases as oppose to the increase in probability of failure. Manglik & Ram [13] studied behavioral analysis of a hydroelectric production power plant under reworking scheme. Singh et al [14] studied the complex systems having three units-super priority, priority and ordinary under pre-emptive resume repairs policy.

Consecutive k-out-of-n systems have been studied by various researchers. Ramamuthy [2] studied reliability of a consecutive-k-out-of-n: F system consists of n ordered components along a line or circle such extensively applied in many field of engineering like power plants, airplane model industrial organizations. Earlier the researchers including [1, 4, 20, 23] studied the complex system by considering the fact that the failed system may be repaired by general repair and they studied the reliability characteristic of a complex system under the fact that only one repair can be employed between two transition states, but there are many situations where more than one repair can be possible between two transition states, when such type possibility exists the system is studied using copula [15].

In continuation to the study of complex systems the authors Singh et al [16] proposed the work with the system which consists of two subsystems with controllers and deliberately human failure. The computer networking has become an essential requirement for most of the industries and organizations. Consequence to the study of networking systems Rawal et al [17] studied modeling and availability analysis of internet data center with various maintenance policies using copula. Rawal et al [18] studied the reliability measures of a local area network via copula linguistics approach. Singh
et al [22] studied availability and cost analysis of a complex system under pre-emptive resume repair policy using copula distribution. Furthermore, a number of researchers dealt with the problems of reliability field but still more attention is required.

In this paper, we have considered mathematical model of a system, which has three units and works under 2-out-of-3: G; policy. Initially in state S0, the system is in perfect state where all three units, switch is in good working condition. Whenever system starts functioning and if first, second and third unit of system fail then the system will approach to S1, S2 and S3 state, respectively. Further, failure in any unit in the system will lead it to complete failure mode i.e. S4, S5 and S6 states. The state S7 and S8 indicates the switch and human failure which is assumed to completely damage the system. Now under consideration the system for 2-out-of-3: G; policy it is clear that states S1, S2 and S3 are in partially failed states, which may be repaired by employing general repair policy. Though the states S4, S5 and S6 are completely failed states, but general repair has already been assigned; therefore, these states will be repaired using general repair policy. The states S7 and S8 are completely damaged states due to which the functioning of entire system shutdown; therefore, these states must be repaired using copula distribution. The system is studied by using supplementary variable technique and Laplace transforms, and various measures of reliability have been discussed and some particular cases are also taken to highlight the result.

The description of state transition diagram of model, assumptions and notation used are given in § 2. In § 3, the mathematical modeling and solution of formulated model are presented. The analytical part, in which the various parameters like availability, MTTF and profit analysis have been evaluated for different values of parameters are given in §4. The results & discussion and conclusions are given in § 4 and 5, respectively.

### 1.1 State Description

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0</td>
<td>All three units system, connecting switch, is in good working condition and no human failure arises in this state. The system is in perfect state.</td>
</tr>
<tr>
<td>S1</td>
<td>In state S1 first unit of system fails with the failure rate λ1. The repair has been assigned to the failed unit and the system is in operational mode with partial failure.</td>
</tr>
<tr>
<td>S2</td>
<td>In state S2 first unit of system fails with the failure rate λ2. The repair has been assigned to the failed unit and the system is in operational mode with partial failure.</td>
</tr>
<tr>
<td>S3</td>
<td>In state S3 first unit of system fails with the failure rate λ3. The repair has been assigned to the failed unit and the system is in operational mode with partial failure.</td>
</tr>
<tr>
<td>S4</td>
<td>In state S4 the system is in complete failure mode, general repair is employed to the previously failed unit, when the unit will be repaired it will start working.</td>
</tr>
<tr>
<td>S5</td>
<td>In state S5 the system is in complete failure mode, general repair is employed to the previously failed unit, when the unit will be repaired it will start working.</td>
</tr>
<tr>
<td>S6</td>
<td>In state S6 the system is in complete failure mode, general repair have been employed to the previously failed unit, when the unit will be repaired, it will start working.</td>
</tr>
</tbody>
</table>
1.2 Assumption

The following assumptions are taken throughout the discussion of model:

1. Initially the system is in perfect state $S_0$ and all units are in good working condition.
2. The system working under 2-out-of-3: G; Policy therefore till the time, two of its unit are in good condition it will work and fulfill the assignment.
3. System fails if more than two units fail.
4. Human failure as well as switch failure completely fails the system.
5. Only one change is allowed at a time in the transitions.
6. Partial failure is repaired by general time distribution.
7. Human failure, and switch failure in the system need fast repairing and hence is repaired by using (Gumbel-Hougaard) family copula.
8. Repaired system works like a new and repair did not damage anything.

1.3 State Transition Diagram of Model

![State Transition Diagram of Model](image-url)

Fig. 1 State Transition Diagram of model
1.4 Notations

| \( \lambda_1 / \lambda_2 / \lambda_3 \) | Failure rate for first unit/second unit/ third unit of system. |
| \( \lambda_3 / \lambda_h \) | Failure rate for switch/ human failure. |
| \( \Phi_i(x) \) | Repair rate of \( i \)th unit of system, \( i=1, 2, 3. \) |
| \( P_0(t) \) | State transition probabilities of system in perfect state \( S_0 \). |
| \( P_i(x,t) \): | State transition probability that the system is in state \( P_i(x,t) \), system is under repair with repair variable \( x, t \). |
| \( C_\theta(u_1,u_2(x)) \): | The expression for joint probability distribution (failed state \( S_i \) to good state \( S_0 \)) according to Gumbel-Hougaard family is given as: \( C_\theta(u_1,u_2(x)) = \mu_0(x) = \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}. \) Where, \( u_1 = \phi(x) \), and \( u_2 = e^x \), \( 1 \leq \theta \leq \infty \) |
| \( E_p(t) \): | Expected profit in interval \([0,t)\) |

2 Formulation of Mathematical Model:

By probability considerations and continuity arguments, we can obtain the following set of difference differential equations governing the present mathematical model:

\[
\begin{align*}
\frac{\partial}{\partial t} + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_h + \lambda_S \quad &P_0(t) = \int_0^\infty \phi_1(x)P_1(x,t)dx \\
+ \int_0^\infty \phi_2(x)P_2(x,t)dx + \int_0^\infty \phi_3(x)P_3(x,t)dx + \int_0^\infty \mu_0(x)P_7(x,t)dx + \int_0^\infty \mu_0(x)P_8(x,t)dx (2.1) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_2 + \lambda_3 + \lambda_h + \lambda_S + \phi_1(x) \quad &P_1(x,t) = 0 (2.2) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_1 + \lambda_3 + \lambda_h + \lambda_S + \phi_2(x) \quad &P_2(x,t) = 0 (2.3) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_2 + \lambda_1 + \lambda_h + \lambda_S + \phi_3(x) \quad &P_3(x,t) = 0 (2.4) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi_1(x) \quad &P_4(x,t) = 0 (2.5) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi_1(x) \quad &P_5(x,t) = 0 (2.6) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi_2(x) \quad &P_6(x,t) = 0 (2.7) \\
\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) \quad &P_7(x,t) = 0 (2.8)
\end{align*}
\]
Initial conditions:

\[ \left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) \right] P_8(x,t) = 0 \]  
(2.9)

\[ P_1(0,t) = \lambda_1 P_0(t) \]  
(2.10)

Boundary conditions:

\[ P_2(0,t) = \lambda_2 P_1(0,t) + \int_0^\infty \phi_1(x) P_5(x,t) dx \]  
(2.11)

\[ P_3(0,t) = \lambda_3 P_1(0,t) + \int_0^\infty \phi_1(x) P_4(x,t) dx + \int_0^\infty \phi_2(x) P_6(x,t) dx \]  
(2.12)

\[ P_4(0,t) = \lambda_2 P_1(0,t) + \lambda_1 P_3(0,t) \]  
(2.13)

\[ P_5(0,t) = \lambda_3 P_1(0,t) + \lambda_1 P_2(0,t) \]  
(2.14)

\[ P_6(0,t) = \lambda_3 P_2(0,t) + \lambda_2 P_3(0,t) \]  
(2.15)

\[ P_7(0,t) = \lambda_2 P_1(0,t) + \lambda_1 P_2(0,t) + \lambda_1 P_1(0,t) + \lambda_1 P_3(0,t) \]  
(2.16)

\[ P_8(0,t) = \lambda_3 P_2(0,t) + \lambda_2 P_3(0,t) + \lambda_1 P_1(0,t) + \lambda_1 P_2(0,t) \]  
(2.17)

Initial conditions:

\[ P_0(0) = 1, \text{ other state probabilities are zero at } t = 0 \]

3 Solution of the model

Taking Laplace transforms of equations (2.1) - (2.17) and using equation (3.1) we obtain.

\[ (s + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5) \tilde{P}_0(s) = [1 + \int_0^{\infty} \phi_1(x) \tilde{P}_1(x,s) dx + \int_0^{\infty} \phi_2(x) \tilde{P}_2(x,s) dx \]

\[ + \int_0^{\infty} \phi_3(x) \tilde{P}_3(x,s) dx + \int_0^{\infty} \mu_0(x) \tilde{P}_7(x,s) dx + \int_0^{\infty} \mu_0(x) \tilde{P}_8(x,s) dx] \]  
(3.1)

\[ \left[ s + \frac{\partial}{\partial x} + \lambda_2 + \lambda_3 + \lambda_4 + \phi_1(x) \right] \tilde{P}_1(x,s) = 0 \]  
(3.2)

\[ \left[ s + \frac{\partial}{\partial x} + \lambda_1 + \lambda_3 + \lambda_4 + \phi_2(x) \right] \tilde{P}_2(x,s) = 0 \]  
(3.3)

\[ \left[ s + \frac{\partial}{\partial x} + \lambda_1 + \lambda_2 + \lambda_4 + \phi_3(x) \right] \tilde{P}_3(x,s) = 0 \]  
(3.4)

\[ \left[ s + \frac{\partial}{\partial x} + \phi_1(x) \right] \tilde{P}_4(x,s) = 0 \]  
(3.5)
Solving (3.2) - (3.9) with the help of (3.10) - (3.24), one may get

\[
\left[ s + \frac{\partial}{\partial x} + \phi_1(x) \right] \bar{P}_3(x,s) = 0
\]

(3.6)

\[
\left[ s + \frac{\partial}{\partial x} + \phi_2(x) \right] \bar{P}_6(x,s) = 0
\]

(3.7)

\[
\left[ s + \frac{\partial}{\partial x} + \mu_0(x) \right] \bar{P}_7(x,s) = 0
\]

(3.8)

\[
\left[ s + \frac{\partial}{\partial x} + \mu_0(x) \right] \bar{P}_8(x,s) = 0
\]

(3.9)

Laplace Transform of boundary conditions:

\[
\bar{P}_1(0,s) = \lambda_4 \bar{P}_0(s)
\]

(3.10)

\[
\bar{P}_2(0,s) = \lambda_2 \bar{P}_0(s) + \int_0^\infty \phi_1 \bar{P}_3(x,s) dx
\]

(3.11)

\[
\bar{P}_3(0,s) = \lambda_2 \bar{P}_0(s) + \int_0^\infty \phi_1 \bar{P}_4(x,s) dx + \int_0^\infty \phi_2 \bar{P}_6(x,s) dx
\]

(3.12)

\[
\bar{P}_4(0,s) = \lambda_2 \bar{P}_1(0,s) + \lambda_4 \bar{P}_3(0,s)
\]

(3.13)

\[
\bar{P}_5(0,s) = \lambda_3 (\bar{P}_1(0,s) + \lambda_4 \bar{P}_2(0,s))
\]

(3.14)

\[
\bar{P}_6(0,s) = \lambda_3 (\bar{P}_2(0,s) + \lambda_2 \bar{P}_3(0,s))
\]

(3.15)

\[
\bar{P}_7(0,s) = \lambda_5 (\bar{P}_0(s) + \bar{P}_2(0,s) + \bar{P}_1(0,s) + \bar{P}_3(0,s))
\]

(3.16)

\[
\bar{P}_8(0,s) = \lambda_8 (\bar{P}_0(s) + \bar{P}_2(0,s) + \bar{P}_1(0,s) + \bar{P}_3(0,s))
\]

(3.17)

Solving (3.2) - (3.9) with the help of (3.10) - (3.24), one may get

\[
\bar{P}_0(s) = \frac{1}{D(s)}
\]

(3.18)

\[
\bar{P}_1(s) = \frac{\lambda_1 (1 - S_{\theta_1}(s + \lambda_2 + \lambda_3 + \lambda_h + \lambda_S))}{D(s)}
\]

(3.19)

\[
\bar{P}_2(s) = \frac{\lambda_2 (1 - S_{\theta_2}(s + \lambda_1 + \lambda_3 + \lambda_h + \lambda_S))}{D(s)}
\]

(3.20)

\[
\bar{P}_3(s) = \frac{\lambda_3 (1 - S_{\theta_3}(s + \lambda_1 + \lambda_2 + \lambda_h + \lambda_S))}{D(s)}
\]

(3.21)

\[
\bar{P}_4(s) = \frac{\lambda_4 (1 - S_{\theta_4}(s))}{D(s)}
\]

(3.22)

\[
\bar{P}_5(s) = \frac{\lambda_5 (1 - S_{\theta_5}(s))}{D(s)}
\]

(3.23)

\[
\bar{P}_6(s) = \frac{\lambda_6 (1 - S_{\theta_6}(s))}{D(s)}
\]

(3.24)
\[
\bar{P}_7(s) = \frac{\bar{P}_2(0,s)(1-S_{\mu_0}(s))}{D(s)} \tag{3.25}
\]
\[
\bar{P}_8(s) = \frac{\bar{P}_3(0,s)(1-S_{\mu_0}(s))}{D(s)} \tag{3.26}
\]
\[
D(s) = \left[ (s + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_h + \lambda_s) - (\lambda_1 S_{\phi_1}(s + \lambda_2 + \lambda_3 + \lambda_h + \lambda_s) + \bar{P}_2(0,s) S_{\phi_2}(s + \lambda_1 + \lambda_3 + \lambda_h + \lambda_s)) + \bar{P}_3(0,s) S_{\phi_3}(s + \lambda_1 + \lambda_2 + \lambda_h + \lambda_s) + \bar{P}_7(0,s) S_{\mu_0}(s) + \bar{P}_8(0,s) S_{\mu_3}(s) \right] \tag{3.27}
\]

where,

\[
\bar{P}_2(0,s) = A_1 \bar{P}_0(s), \quad \bar{P}_3(0,s) = \left( \frac{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_h + \lambda_s}{B_1} \right) \bar{P}_0(s) = B_2 \bar{P}_0(s), \quad \bar{P}_4(0,s) = \lambda_1 B_3 \bar{P}_0(s), \\
\bar{P}_5(0,s) = \lambda_1(\lambda_2 + A_1) \bar{P}_0(s), \quad \bar{P}_6(0,s) = (\lambda_3 A_1 + \lambda_2 B_2) \bar{P}_0(s), \quad \bar{P}_7(0,s) = \lambda_s(1 + \lambda_1 + A_1 + B_2) \bar{P}_0(s), \quad \bar{P}_8(0,s) = \lambda_h(1 + \lambda_1 + A_1 + B_2) \bar{P}_0(s).
\]

\[
A_1 = \frac{\lambda_2 + \lambda_3 + \lambda_1 \phi_1}{1 - \frac{\lambda_1 \phi_1}{s + \phi_1}}, \quad B_1 = \left( 1 - \frac{\lambda_1 \phi_1}{s + \phi_1} - \frac{\lambda_2 \phi_2}{s + \phi_2} \right), \quad B_2 = \left( \frac{\lambda_3 + \lambda_1 \phi_1 + \lambda_2 \phi_2}{B_1} \right), \quad B_3 = \lambda_2 + B_2.
\]

The Laplace transforms of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

\[
\bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s) + \bar{P}_3(s) \tag{3.28}
\]
\[
\bar{P}_{failed}(s) = \bar{P}_4(s) + \bar{P}_5(s) + \bar{P}_6(s) + \bar{P}_7(s) + \bar{P}_8(s) \tag{4.2}
\]

4 Particular cases:

4.1 Availability Anaysis

4.2 When repair follows exponential distribution.

Setting \( \bar{S}_{\mu_0}(s) = \frac{\mu_0}{s + \mu_0}, \bar{S}_{\phi_i}(s) = \frac{\phi_i}{s + \phi_i}, i = 1, 2, 3 \) in equation (3.28) and \( \mu_0(x) = \exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta} \)

(1) Setting the values of different parameters as \( \lambda_1 = 0.03, \lambda_2 = 0.032, \lambda_3 = 0.025, \lambda_s = 0.035, \lambda_h = 0.025, \)
\( = 1, \theta = 1, x = 1 \), then taking inverse Laplace transform, one can obtain,

\[
P_{up}(t) = -0.01014834 e^{-1.117000 t} + 0.0043108 e^{-1.09000 t} + 0.0023228 e^{2.784313 t} - 0.0215549 e^{-1.141355 t} - 0.0028393 e^{-1.097352 t} + 0.00001238 e^{-1.000457 t} + 0.0018635 e^{-0.986884 t} + 1.00512796 e^{-0.000967 t} \tag{4.1}
\]

For, \( t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90... \). One, may get different values of \( P_{up}(t) \) as shown in Table 1.
Table 1. Time vs. Availability.

<table>
<thead>
<tr>
<th>Time(t)</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>10</td>
<td>0.996</td>
</tr>
<tr>
<td>20</td>
<td>0.986</td>
</tr>
<tr>
<td>30</td>
<td>0.976</td>
</tr>
<tr>
<td>40</td>
<td>0.967</td>
</tr>
<tr>
<td>50</td>
<td>0.958</td>
</tr>
<tr>
<td>60</td>
<td>0.949</td>
</tr>
<tr>
<td>70</td>
<td>0.939</td>
</tr>
<tr>
<td>80</td>
<td>0.930</td>
</tr>
<tr>
<td>90</td>
<td>0.921</td>
</tr>
<tr>
<td>100</td>
<td>0.913</td>
</tr>
</tbody>
</table>

Fig. 2. Variation of Availability with respect to time.

4.3 Reliability Analysis:

Taking all repairs to zero in equation (3.28) and then taking inverse Laplace transform, for given failure rates $\lambda_1=0.03$, $\lambda_2=0.032$, $\lambda_3=0.025$, $\lambda_c = 0.035$, $\lambda_b = 0.025$ one can get the expression for reliability:

$$ R(t) = -2.0312500 e^{(-0.1470000 \cdot t)} + e^{(-0.0117000 \cdot t)} + 1.0312550 e^{(-0.1150000 \cdot t)} + e^{(-0.1220000 \cdot t)} $$(4.2)
4.4 C. Mean time to failure (MTTF)

Setting

\[ \bar{S}_{\mu_0}(s) = \frac{\mu_0}{s + \mu_0}, \quad \bar{S}_{\phi_i}(s) = \frac{\phi_i}{s + \phi_i}, \quad i = 1, 2, 3, \quad \mu_0(x) = \exp[x^\theta + \{\log\phi(x)\}^\theta]^1/\theta \]

Taking all repairs to zero in equation (3.28), and taking limit \( s \) tends to zero one can obtain the expression for MTTF as:

\[
M.T.T.F = \frac{1}{(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_S + \lambda_h)} \left[ 1 + \frac{\lambda_1}{(\lambda_2 + \lambda_3 + \lambda_S + \lambda_h)} + \frac{\lambda_2}{(\lambda_1 + \lambda_3 + \lambda_S + \lambda_h)} + \frac{\lambda_3}{(\lambda_1 + \lambda_2 + \lambda_S + \lambda_h)} \right]
\]

Setting \( \lambda_1 = 0.03, \lambda_2 = 0.032, \lambda_3 = 0.025, \lambda_S = 0.035, \lambda_h = 0.025 \) and varying \( \lambda_1 \) as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (4.3) one may obtain Table 2 whose column 2 demonstrates variation of MTTF with respect to \( \lambda_1 \).

Setting \( \lambda_1 = 0.03, \lambda_2 = 0.03, \lambda_3 = 0.025, \lambda_S = 0.035, \lambda_h = 0.025 \) and varying \( \lambda_2 \) as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (4.3) one may obtain Table 2 whose column 3 demonstrates variation of MTTF with respect to \( \lambda_2 \).

Setting \( \lambda_1 = 0.03, \lambda_2 = 0.032, \lambda_3 = 0.025, \lambda_S = 0.035, \lambda_h = 0.025 \) and varying \( \lambda_3 \) as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (4.3) one may obtain Table 2, whose column 4 shows variation of MTTF with respect to \( \lambda_3 \).

Setting \( \lambda_1 = 0.03, \lambda_2 = 0.032, \lambda_3 = 0.025, \lambda_S = 0.035, \lambda_h = 0.025 \) and varying \( \lambda_S \) as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (4.3) one may obtain Table 2, which reveals variation of MTTF with respect to \( \lambda_S \) in column 5.

Setting \( \lambda_1 = 0.03, \lambda_2 = 0.032, \lambda_3 = 0.025, \lambda_S = 0.035 \) and varying \( \lambda_h \) as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (4.3) one may obtain Table 2, which reveals variation of MTTF with respect to \( \lambda_h \) in column 6.
### Table 2: Variation of MTTF with respect to failure rates.

<table>
<thead>
<tr>
<th>Failure rate</th>
<th>MTTF $\lambda_1$</th>
<th>MTTF $\lambda_2$</th>
<th>MTTF $\lambda_3$</th>
<th>MTTF $\lambda_S$</th>
<th>MTTF $\lambda_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>1.656</td>
<td>1.656</td>
<td>1.655</td>
<td>2.514</td>
<td>3.306</td>
</tr>
<tr>
<td>0.02</td>
<td>1.652</td>
<td>1.653</td>
<td>1.650</td>
<td>2.454</td>
<td>3.205</td>
</tr>
<tr>
<td>0.03</td>
<td>1.648</td>
<td>1.649</td>
<td>1.646</td>
<td>2.398</td>
<td>3.111</td>
</tr>
<tr>
<td>0.04</td>
<td>1.645</td>
<td>1.645</td>
<td>1.642</td>
<td>2.344</td>
<td>3.021</td>
</tr>
<tr>
<td>0.05</td>
<td>1.641</td>
<td>1.642</td>
<td>1.638</td>
<td>2.292</td>
<td>2.937</td>
</tr>
<tr>
<td>0.06</td>
<td>1.634</td>
<td>1.639</td>
<td>1.635</td>
<td>2.243</td>
<td>2.857</td>
</tr>
<tr>
<td>0.07</td>
<td>1.631</td>
<td>1.636</td>
<td>1.631</td>
<td>2.196</td>
<td>2.781</td>
</tr>
<tr>
<td>0.08</td>
<td>1.628</td>
<td>1.633</td>
<td>1.628</td>
<td>2.150</td>
<td>2.709</td>
</tr>
<tr>
<td>0.09</td>
<td>1.625</td>
<td>1.630</td>
<td>1.625</td>
<td>2.106</td>
<td>2.641</td>
</tr>
</tbody>
</table>

![Fig. 4](image.png)

**Fig. 4** The variation of Failure rate verses. MTTF.

### 5 Cost analysis

Let the failure rates of the system be $\lambda_1=0.030$, $\lambda_2=0.032$, $\lambda_3=0.025$, $\lambda_S=0.025$, $\lambda_h=0.035$, mean time to repair be $\phi(x)=1$, and $x=1$, $\theta=1$, $\phi(x)=1$, $\bar{S}_\mu(s)=\frac{\mu}{s+\mu}$, $\bar{\phi}_i(s)=\frac{\phi_i}{s+\phi_i}$, $i=1, 2, 3$ in equation (3.28) and then taking inverse Laplace transform, one can obtain (5.1). Let the service facility be always available, then expected profit during the interval $[0, t)$ is

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$$
Where $K_1$ and $K_2$ are revenue service cost per unit time. Hence

$$E_p(t) = K_1(0.009085355 e^{(-1.117000 \cdot t)} - 0.003954886 e^{(-1.090000 \cdot t)} - 0.00834241 e^{(-2.784313 \cdot t)} + 0.018885407 e^{(-1.1413558 \cdot t)} + 0.002587397 e^{(-1.097353 \cdot t)} - 0.0000123246 e^{(-1.0045700 \cdot t)} - 0.00188830 e^{(-0.9868837 \cdot t)} - 1039.8293 e^{(-0.000966627 \cdot t)} + 1039.813) - K_2 \cdot t$$

(5.1)

Setting $K_1 = 1$ and $K_2 = 0.50, 0.40, 0.30, 0.20, \text{and} 0.10$, respectively and varying $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, \ldots$, one get Table 4.

Table 3. The values of Expected Profit ($E_p(t)$) for different values of $K_2$ for different value of Time.

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>$E_p(t)$; $K_2=0.5$</th>
<th>$E_p(t)$; $K_2=0.4$</th>
<th>$E_p(t)$; $K_2=0.30$</th>
<th>$E_p(t)$; $K_2=0.20$</th>
<th>$E_p(t)$; $K_2=0.10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>4.987</td>
<td>5.987</td>
<td>6.987</td>
<td>7.987</td>
<td>8.987</td>
</tr>
<tr>
<td>20</td>
<td>9.893</td>
<td>11.893</td>
<td>13.893</td>
<td>15.893</td>
<td>17.893</td>
</tr>
<tr>
<td>40</td>
<td>19.421</td>
<td>23.422</td>
<td>27.421</td>
<td>31.421</td>
<td>35.421</td>
</tr>
<tr>
<td>50</td>
<td>24.045</td>
<td>29.045</td>
<td>34.045</td>
<td>39.045</td>
<td>44.045</td>
</tr>
<tr>
<td>60</td>
<td>28.576</td>
<td>34.576</td>
<td>40.576</td>
<td>46.576</td>
<td>52.576</td>
</tr>
<tr>
<td>70</td>
<td>33.015</td>
<td>40.015</td>
<td>47.015</td>
<td>54.015</td>
<td>61.015</td>
</tr>
<tr>
<td>80</td>
<td>37.363</td>
<td>45.363</td>
<td>53.364</td>
<td>61.363</td>
<td>69.363</td>
</tr>
<tr>
<td>90</td>
<td>41.622</td>
<td>50.622</td>
<td>59.622</td>
<td>68.622</td>
<td>77.622</td>
</tr>
<tr>
<td>100</td>
<td>45.791</td>
<td>55.791</td>
<td>65.791</td>
<td>75.791</td>
<td>85.791</td>
</tr>
</tbody>
</table>

Fig. 5 The variation of Expected Profit vs. Time for different values of $K_2$. 


6 Result and Discussion:

The availability of the complex repairable system changes with respect to the time, when failure rates are fixed at different values (see Table 1 and Fig. 2). When failure rates are fixed at lower values i.e., \( \lambda_1 = 0.030, \lambda_2 = 0.032, \lambda_3 = 0.025, \lambda_h = 0.025 \) and \( \lambda_S = 0.035 \), availability of the system decreases and probability of failure increase, with passage of time and ultimately becomes steady to the value zero after a sufficient long interval of time. Hence, one can safely predicts the future behavior of complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model (see Fig. 2).

The information of reliability of system, when no repair is employed for the system is shown in Figure 3. The reliability of system is less than the availability of system, which indicates necessity of employing repair for in repairable system (see Fig. 3).

The values of MTTF of the system for failure rate \( \lambda_1, \lambda_2, \lambda_3, \lambda_S, \) and \( \lambda_h \) are listed in Table 2 and its variation is shown in Fig. 4, which shows that the MTTF decreases as failure rate increases. The MTTF corresponding to failure rates \( \lambda_1, \lambda_2, \lambda_3 \) are very much close to each other; but for \( \lambda_S \) and \( \lambda_h \) values the MTTF are quite different and is comparatively higher than for \( \lambda_1, \lambda_2, \lambda_3 \). This indicates that the failure rates \( \lambda_S \) and \( \lambda_h \) are more responsible for proper operation of the system.

7 Conclusions:

Present study clearly indicate that for a given sets of parametric values of failure rates, the availability and reliability of system decreases with the time. Availability values are greater as compared to the values of reliability for uniform change of time variable. It indicates that the repairable system is more reliable as compared to non-repairable systems. The MTTF; corresponding to the failure rates \( \lambda_S \) and \( \lambda_h \); is comparatively high; hence it is more responsible for proper functioning of system.

When revenue cost per unit time \( K_1 \) fixed at 1, service cost \( K_2 = 0.5, 0.40, 0.30, 0.20, 0.10 \), profit has been calculated and results are demonstrated by graphs in figure 5. One can observe that as the service cost decreases profit increases.

Researchers can further discuss like comparative study of copula for the particular system. This system can analyze by help of other types of copula like Archimedean copula, Clayton copula, and Franklin copula. Sensitivity analysis of the system is left for future research.

References


