Analysis and simulation of adaptive control strategies for uncertain bio-inspired sensor systems

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Abstract. We consider mechanical systems, that are not known precisely. In detail, we assume that the system parameters are unknown (uncertainty of system). Moreover, we allow for uncertainties with respect to perturbations from the environment (ground excitations). Such (non-linearly perturbed) systems need adaptive controllers which achieve a pre-defined control objective. Almost all known controllers offer the same drawback: (possibly unboundedly) monotonically increasing gain parameters. For application it is necessary to design improved adaptation laws. We present numerical simulations of several models primarily in application to a bio-inspired sensor model.

1 Introduction

This section informs about the biological background of the systems under investigation, their modeling and treatment.

1.1 Biological motivation

In nature there are various senses that allow animals to perceive their environment. Among them are the senses of touch, of which the sense of vibration is a special case. Vibrations are an important piece of environmental information that insects rely on, especially arachnids, such as spiders and scorpions. To perceive vibrations, they have different types of sensilla, see [2] and [24]. Vertebrates may also possess the sense of vibration, such as cats, rats and sea lions. They can perceive vibrations with the help of their vibrissae, see Fig. 1.
Although these biological vibration sensors have a different physiology, see [19], [23], they share common properties. When in touch with an oscillating object, they are stimulated by mechanical oscillation energy, which is then transmitted to the receptor cells. The sensibility of these cells is continuously adjusted so that the receptor system converges to the rest position, despite the continued excitation. This means that the receptors are in a permanent state of adaptation and the continuous excitation is damped. Therefore, the information about the continuous excitation is considered irrelevant, once it has been perceived. If however a different excitation, such as a sudden deviation of the vibrissa sensor, occurs, this information is relevant and the sensor has to be sensitive to perceive it. If, for example, a cat is exposed to wind, the recognition of the resulting excitation of the whiskers will be damped and ignored. If the cat encounters an obstacle, the receptors should still be sensitive enough to perceive the sudden deviation of the whiskers while the wind excitation persists. Therefore, the adaption process has to include the possibility of not only increasing, but also decreasing the damping of the receptor system and therefore ensuring sensitivity.

1.2 Modeling

Motivated by these biological observations, a receptor is modeled as a spring-mass-damper system, see Fig. 2, for further considerations. It consists of a mass $m$ under the action of an internal force $u(t)$ within a rigid frame that is forced by an unknown external time dependent excitation $x_0(t)$.
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- $x(t)$ – absolute coordinate,
- $y(t)$ – relative coordinate (system output),
- $m$ – mass,
- $d$ – viscous damping factor,
- $c$ – spring stiffness,
- $x_0(t)$ – indirect excitation (spatial disturbance),
- $u(t)$ – control force.

The system is excited by an unknown time-varying displacement $x_0(t)$. Applying Newton’s second law, the differential equation (ODE) governing the system can be obtained:

$$m \ddot{y}(t) + d \dot{y}(t) + c y(t) + m \ddot{x_0}(t) = u(t).$$

(1.1)

The goal is to achieve a predefined movement of the mass $m$, such as tracking a set point trajectory $y_{ref}$ or stabilization. It is obvious that the sole possibility of influencing the system lies in the control force $u(t)$. Hence, a controller has to be implemented to act on the system in a way that the desired system output is generated. The objective of this contribution is to find a suitable control strategy which can deal with the specialties of this biologically inspired sensory system.

1.3 Scope of this contribution

It is important to point out that all system parameters are supposed to be unknown or uncertain, i.e., that we do not know the parameters, but only parameter intervals. This is due to the the sophisticated nature of the biological system. The correct modeling of the system parameters would be a very complex task. This is why traditional control methods fail, as they rely on the knowledge of those parameters. Adaptive controllers are a way of dealing with this control problem. Previous works on this has been conducted in [1], [11], [3], [27], [10], [20], [6], and [7]. Their work is continued and extended in this contribution.

In the following we will analyze the systems properties and present common control strategies from literature. We point out the drawbacks and present a series of improved controllers (feedback law plus adaptor). Each step is numerically analyzed for advantages and disadvantages which then imply the next step of improvement. The working principle of each controller is shown in numerical simulations which prove that the controller in fact works effectively.

Except of the classical controller from literature, the improved controllers are not yet founded by a stringent theory. But all simulation results encourage one to continue these investigations, in particular in application to more evolved sensor systems.

2 Controller design: adaptive control

2.1 Control objectives

There are three main goals to be achieved by the controllers that are discussed here:

- $\lambda$-tracking,
- $\lambda$-stabilization and
- stabilization.

Since we deal with non-linearly perturbed systems (due to the existence of the ground excitation), which are not necessarily autonomous, particular attention is paid to $\lambda$-tracking. The principle behind $\lambda$-tracking is to design a universal controller that uses information obtained from the system’s behavior in order to automatically adjust its parameters so that the system tracks a given reference signal.
$y_{\text{ref}}(\cdot)$ with a prescribed accuracy $\lambda$. The parameter $\lambda > 0$ denotes the size of the tolerated tracking error.

Precisely, given $\lambda > 0$, a control strategy $y \mapsto u$ is sought which, when applied to this system, achieves $\lambda$-tracking for every reference signal $y_{\text{ref}}(\cdot)$ of a certain class $R$ (a Sobolev space $W^{2,\infty}$), i.e., the following:

(i) every solution of the closed-loop system is defined and bounded on $\mathbb{R}_{\geq 0}$, and
(ii) the output $y(\cdot)$ tracks $y_{\text{ref}}(\cdot)$ with asymptotic accuracy quantified by $\lambda > 0$ in the sense that

$$\max \left\{ 0, \| y(t) - y_{\text{ref}}(t) \| - \lambda \right\} \to 0 \text{ as } t \to +\infty.$$  

The last condition means that the system output $y(t)$ is forced, via an adaptive feedback mechanism, into the $\lambda$-tube around the set point trajectory $y_{\text{ref}}$, see Fig. 3 ([3]). Hence, a certain error value $\| e(t) \| := \| y(t) - y_{\text{ref}}(t) \| \leq \lambda$ is accepted when the controller is in operation. $\lambda$-stabilization is only a special case of tracking with $y_{\text{ref}} \equiv 0$ and $\lambda > 0$, whereas stabilization is of the type $y_{\text{ref}} \equiv 0$ and $\lambda = 0$. By choosing $\lambda$, the acceptable error value can be specified. $\lambda$-tracking allows simple feedback mechanisms.

2.2 Controller structure

When designing a controller, the feedback structure and the control parameters have to be determined. Due to stability reasons, a PD-feedback structure is required. The question of how to choose the proportional and derivative gains arises.

In classical control engineering, the control parameters can be determined by a number of different methods (e.g., the Bode plot, the root locus plot or the pole-zero plot). However, all of these methods require the precise knowledge of the plant’s parameters to determine the control gains.

In the case that is discussed in this contribution, all system parameters are supposed to be unknown. Hence, it is necessary to employ a control strategy that can adapt to different system parameters and still achieve the desired goal. For this purpose, adaptive controllers are chosen. First, consider the classic PD-controller:

$$e(t) := y(t) - y_{\text{ref}}(t)$$
$$u(t) = -k_1 e(t) - k_2 \dot{e}(t)$$

with $k_1, k_2 \in \mathbb{R}$, const.
Remark 1. The spring-mass-damper system (1.1) with $\ddot{x}_0 \equiv 0$ is also stable without any D-feedback. This is due to the fact that with $d > 0$ all of its poles are already located in the open left half complex plane. Therefore, system (1.1) is also stable with just a P-feedback controller, proven in [7].

As discussed above, $k_1$ and $k_2$ cannot be determined by classic methods. To simplify the problem, $k_1$ and $k_2$ are first set equal to $k$. Then, $k$ shall be seen as a function of $t$ in order to influence the error as required:

$$
e(t) := y(t) - y_{\text{ref}}(t)
$$

$$
u(t) = -k(t)e(t) - k(t)\dot{e}(t)
$$

(2.1)

2.3 Adaptors

Since $t \mapsto k(t)$ is still unknown, an adaptor has to be implemented. It has to be designed in such a way that $k(t)$ increases sufficiently to reduce the error value to an acceptable level, but is still small enough to enable the system to have the desired sensitivity. If $k(t)$ was increased intensely, $\lambda$-tracking would be achieved, whereas the system would become unnecessarily stiff and therefore insensitive to other stimuli that are not part of the disturbance. Therefore, the control objective of the closed-loop system affects the adaptor design. A “classical” adaptor is given in [13] and [14]:

$$
k(t) = \left(\max\{0, \|e(t)\| - \lambda\}\right)^2
$$

(2.2)

with $\lambda > 0$. The adaptor (2.2) works as follows:

If the absolute error value $\|e\|$ is greater than $\lambda$, $k$ is being increased by the square of the positive difference $\|e\| - \lambda$. If $\|e\|$ is smaller than $\lambda$, the increase of $k$ is set to zero, so there is no further increase. If the system output leaves the $\lambda$-neighborhood again, $k$ is increased further. This constitutes a dead-zone behavior. Therefore, (2.1) and (2.2) form a basic $\lambda$-tracker from literature, see [3].

2.4 Problem definition

The goal of this contribution is to consider $\lambda$-tracking, as discussed above and shown in Fig. 3. The utilization of a $\lambda$-neighborhood has several advantages:

- a simple control objective,
- simple feedback laws,
- no need for exact tracking.

This will be achieved without any knowledge of the system parameters. First, an existing adaptive controller for this problem is implemented and tested using the software Matlab (by The MathWorks, Version R2008a). Then, various control strategies are designed, simulated and compared with the existing controller. Goals are in detail:

- to do a survey of previous work,
- to implement and simulate existing control strategies,
- to extend and adapt the existing controllers,
- and to find new control strategies and compare them with the existing controllers.

Whenever new controllers are implemented, their performance has to be evaluated for comparison purposes. The controllers are rated by:
length of settling time at which the system output enters the $\lambda$-tube without leaving it immediately again,
level of gain parameters, level of error inside the $\lambda$-tube,
complexity of controller equations.

3 Basic adaptors - State of the art

Adaptive controllers are considered using modifications of the classical adaptor whereas the PD-feedback law is kept unaltered.

3.1 Current control strategy

A preferred control strategy is the classical adaptor together with a PD-feedback, i.e., (2.1) and (2.2):

$$
\begin{align*}
e(t) & := y(t) - y_{\text{ref}}(t), \\
u(t) & = - \left( k(t) e(t) + \kappa \frac{d}{dt}(k(t) e(t)) \right), \\
k(t) & = \left( \max \left\{ 0, \| e(t) \| - \lambda \right\} \right)^2, \quad k(0) = k_0 \in \mathbb{R} \text{ (arbitrarily chosen)},
\end{align*}
$$

with $\lambda > 0$, $y_{\text{ref}}(t), u(t), e(t) \in \mathbb{R}^m$, $k(t) \in \mathbb{R}$ and $\kappa > 0$ preserving dimensions (chosen $\kappa = 1$ in the following).

This controller consists of very simple feedback and adaptation laws, it is only based on the output of the system and does not contain any system parameter. In this sense the controller is universal. It achieves $\lambda$-tracking for every set of system parameters.

Theorems, valid for a generalized system class, and respective proofs can be found in [3]. In the following, we exemplarily present the system class and a theorem:

**Definition 1.** The normalized equations of motion (1.1) fall into the category of quadratic, finite-dimensional, nonlinearly perturbed, $m$-input $u(\cdot)$, $m$-output $y(\cdot)$ systems (MIMO-systems) of strict relative degree two$^1$ of the form

$$
\begin{align*}
\dot{y}(t) & = A_2 \dot{y}(t) + f_1(s_1(t), y(t), z(t)) + Gu(t), \\
\dot{z}(t) & = A_5 z(t) + A_0 \dot{y}(t) + f_2(s_2(t), y(t)) , \\
y(t_0) & = y_0, \quad \dot{y}(t_0) = y_1, \quad z(t_0) = z_0,
\end{align*}
$$

with $y(t), y_0, y_1, u(t) \in \mathbb{R}^m$, $z(t), z_0 \in \mathbb{R}^{n-2m}$, $A_2, G \in \mathbb{R}^{m \times m}$, $A_5 \in \mathbb{R}^{(n-2m)(n-2m)}$, $A_0 \in \mathbb{R}^{(n-2m) \times m}$, $n \geq 2m$, and, for natural number $q_1$ and $q_2$ it holds

(i) $\text{spec}(G) \subset \mathbb{C}_+$, i.e., the spectrum of the “high-frequency gain” lies in the open right-half complex plane;

(ii) $s_1(\cdot) \in L^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^{q_1})$, $s_2(\cdot) \in L^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^{q_2})$ may be thought of as (bounded) disturbance terms, where $s_1(t) = \psi_1(t, y(t), \dot{y}(t), z(t))$ is also possible with $\psi_1(\cdot, \cdot, \cdot, \cdot) \in L^\infty(\mathbb{R}_{\geq 0} \times \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R}^{n-2m}; \mathbb{R}^{q_1});$

$^1$ The strict relative degree of a system indicates which derivative order of $y$ (the system output) the input $u$ directly acts on.
Theorem 1. Let $\lambda > 0$. Then the controller (3.1) applied to any system class of Definition 1 yields, for any reference signal $y_{\text{ref}}(\cdot) \in \mathcal{R}$, any disturbance terms $s_1(\cdot) \in L^\infty(\mathbb{R}_0; \mathbb{R}^q)$ and $s_2(\cdot) \in L^\infty(\mathbb{R}_0; \mathbb{R}^q)$, and any initial data
\[
(y_0, y_1, z_0, k_0) \in \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R}^{n-2m} \times \mathbb{R},
\]
a closed-loop system (a feedback controlled initial-value problem)
\[
\begin{align*}
y(t) &= \zeta(t), \\
\dot{\zeta}(t) &= A_2 \zeta(t) + f_1(s_1(t), y(t), z(t)) - G \left[ k(t) (y(t) - y_{\text{ref}}(t)) + k(t) (\zeta(t) - \dot{y}_{\text{ref}}(t)) \right] \\
&\quad + \max \left\{ 0, \|y(t) - y_{\text{ref}}(t)\| - \lambda \right\}^2 \left( y(t) - y_{\text{ref}}(t) \right), \\
\dot{z}(t) &= A_5 z(t) + A_0 \zeta(t) + f_2(s_2(t), y(t)), \\
\dot{k}(t) &= \max \left\{ 0, \|y(t) - y_{\text{ref}}(t)\| - \lambda \right\}^2,
\end{align*}
\]
(3.3) with
\[
y(0) = y_0, \xi(0) = y_1, z(0) = z_0, k(0) = k_0
\]
whose (maximal) solution
\[
(y, \zeta, z, k) : [0, t') \to \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R}^{n-2m} \times \mathbb{R}
\]
has the following properties:
(i) $t' = \infty$, i.e. there does not exist a finite escape time;
(ii) $\lim_{t \to +\infty} k(t)$ exists and is finite;
(iii) the solution, $\dot{\zeta}(\cdot), \dot{z}(\cdot)$ and $u(\cdot)$, as in (3.1), are bounded;
(iv) $\lim \sup_{t \to +\infty} \|y(t) - y_{\text{ref}}(t)\| \leq \lambda$.

Remark 2. The limit $k_\infty$ in Assertion (ii) of Theorem 1 is unknown and might be unfeasibly large.

Controller (3.1) forms the basis of our numerical investigations and improvements done in the sequel.

Remark 3. We point out, that the adaptive nature of the upcoming controllers is expressed by the arbitrary choice of the system parameters. Obviously numerical simulation needs fixed (and known) system data, but the controllers adjust their gain parameter to each set of system data.

Simulation 1. The theorems state that controller (3.1) works, but the performance is unknown. We choose the following parameters: $m = 1, d = 5, c = 10, \lambda = 0.2, t \to y_{\text{ref}}(t) = 0, t \to \dot{x}_0(t) = 3 \sin(t) + 10 \cos(2t)$. Figure 4 shows the system performance. The gain $k$ increases too slow and as a result, $\lambda$-tracking is not achieved during the depicted simulation time, only for $t \to +\infty$. The gain $k$ has to be increased faster.
3.2 Modifications to the classical adaptor

In this section we introduce some modifications of the control strategy (3.1). These will show up with various adaptors and, later on, with an altered feedback law (only improved in (3.4)). The main feature is that every adaptor to be used in simulations appears as an ODE of first order for $k(\cdot)$, generalizing (2.2). We consider controllers of the general form

$$
\begin{align*}
    e(t) &:= y(t) - y_{ref}(t), \\
    u(t) &= -k(t) \left( e(t) + \kappa \dot{e}(t) \right), \\
    \dot{k}(t) &= f_\lambda(k(t), e(t), t), \quad k(0) = k_0 \in \mathbb{R} \text{ (arbitrarily chosen)},
\end{align*}
$$

(3.4)

with $\lambda > 0$, $\kappa > 0$.

**Gain coefficient $\gamma$**

To increase the speed of adaptation, we introduce a gain coefficient $\gamma$ in the classical adaptor in the following way (see [17], [3]):

**Adaptor 1:**

$$
\dot{k}(t) = \gamma \left( \max \left\{ 0, \|e(t)\| - \lambda \right\} \right)^2
$$

(3.5)

with $\gamma > 1$, $\lambda > 0$.

The refinement of the adaptor (3.5) as opposed to adaptor (2.2) is the introduction of the gain coefficient $\gamma$. It enables $k$ to increase much faster if $\gamma$ is set to a high value. As this setting is not part of the adaptive nature of the controller, it has to be chosen by the designer in advance (see [3]; [27]).

But, a small parameter $\gamma$ does not force the system output into the $\lambda$-strip, hence only an appropriate value of $\gamma$ yields a good tracking performance, the gain parameter tends to a finite limit. Once inside the tube, $y$ never leaves the $\lambda$-tube and therefore $k$ is kept constant. If the value is too large, this results in a low sensitivity of the sensor, which can now be described as “deaf” to certain disturbances. Therefore, further improvements have to be done.

**Variation of exponent**

Consider a generalized version of the adaptor (3.5):
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Adaptor 2: \( \dot{k}(t) = \gamma (\max\{0, \|e(t)\| - \lambda\})^p \),

(3.6)

with \( \lambda > 0, \ p > 0 \) is the adaptor’s exponent. So far, it has been set to \( p = 2 \). This ensures that larger errors have a much greater effect on the increase of \( k \) than smaller errors, so that the trend towards the \( \lambda \)-strip is faster when the system output is quite far away.

However, if \( \|e\| \) is close to the \( \lambda \)-strip and \( 0 < \|e\| - \lambda < 1 \), an exponent of \( p = 2 \) leads to an even smaller number. This means, that an error value close to \( \lambda \) generates only a very slow increase of \( k \), due to the quadratic exponent of the adaptor. To counteract this behavior, \( p \) is varied from \( p = 0.5 \) to \( p = 2 \). On both sides of \( \lambda + 1 \) best and worst effects are obvious.

To include this knowledge into the adaptor, its behavior is made dependent on the value of \( \|e\| \), to achieve a kind of scheduling of \( \dot{k}(t) \):

Adaptor 3: \[
\dot{k}(t) = \begin{cases} 
\gamma (\|e(t)\| - \lambda)^2, & \|e(t)\| \geq \lambda + 1 \\
\gamma (\|e(t)\| - \lambda)^{0.5}, & \lambda \leq \|e(t)\| < \lambda + 1 \\
0, & \|e(t)\| < \lambda 
\end{cases}
\]

(3.7)

with \( \lambda > 0, \ \gamma > 1 \).

The adaptor now increases \( k \) with a quadratic exponent if \( \|e\| \) is far away from the \( \lambda \)-strip, and with an exponent of \( p = 0.5 \), if \( \|e\| \) is greater than but close to \( \lambda \). While \( \|e\| \) is inside the \( \lambda \)-strip, \( k \) is kept constant. Figure 5 shows the system performance with the adaptor (3.7) using the same simulation parameters, except for \( \gamma \). The effect of the new scheduling is obvious: Although the system performance and the transient behavior of both \( y \) and \( k \) are very similar to that of the controller in Fig. ??, a much lower value of \( \gamma \) was necessary to achieve this result (compare Fig. ?? (left) with Fig. 5 (left), and Fig. ?? (right) with Fig. 5 (right)). This is due to the better performance of the exponent \( p = 0.5 \) in proximity to the \( \lambda \)-tube.

The gain parameter increased from \( k^* = 30 \) to \( k^* = 110 \) for \( \gamma = 150 \) and stays constant, although the control objective is achieved more effectively than necessary. It would be advantageous to decrease the gain parameter, if possible. This is introduced in the next subsection.

3.3 Improved classical adaptors

Intensive studies of numerous adaption laws for adaptive controllers have been conducted ([27], [10]). One of the most promising changes discussed there is the improvement by adding a term for the re-
duction of $k$.

It is conceivable to allow $k$ to simply decrease linearly after $y$ has been inside the $\lambda$-tube for a pre-defined time $t_d$. The according extension of Adaptor 3 is:

$$\textbf{Adaptor 4: }\dot{k}(t) = \begin{cases} 
\gamma (||e(t)|| - \lambda)^2, & \lambda + 1 \leq ||e(t)|| \\
\gamma (||e(t)|| - \lambda)^{0.5}, & \lambda \leq ||e(t)|| < \lambda + 1 \\
0, & ||e(t)|| < \lambda \land t - t_e < t_d \\
-\sigma, & ||e(t)|| < \lambda \land t - t_e \geq t_d
\end{cases}$$

(3.8)

with $\lambda > 0, \gamma > 1, \sigma > 0, t_d > 0$.

The linear reduction of $k$ is followed by an increase, as soon as $y$ has left the $\lambda$-tube. However, the decrease of $k$ is quite slow, which can be overcome by increasing $\sigma$. The introduction of a linear reduction term also results in a globally fixed decrease rate of $k$. It might be advantageous to have a more drastic decrease if $k$ is at a high level, and a slower one if $k$ is already small.

To improve the adaptor (3.8), the term for the reduction of $k$ is altered to be a function of $k$ (see [10]):

$$\textbf{Adaptor 5: }\dot{k}(t) = \begin{cases} 
\gamma (||e(t)|| - \lambda)^2, & \lambda + 1 \leq ||e(t)|| \\
\gamma (||e(t)|| - \lambda)^{0.5}, & \lambda \leq ||e(t)|| < \lambda + 1 \\
0, & ||e(t)|| < \lambda \land t - t_e < t_d \\
-\sigma k(t), & ||e(t)|| < \lambda \land t - t_e \geq t_d
\end{cases}$$

(3.9)

with $\lambda > 0, \gamma > 1, \sigma > 0, t_d > 0$.

The term $-\sigma k(t)$ enables the controller to reduce the gain exponentially. As before, the system output has to stay inside the $\lambda$-tube for a pre-specified time $t_d$, while $k$ is kept constant ($\dot{k} = 0$). After that time has passed, the reduction starts. When the $\lambda$-tube is left, the usual growth of $k$ starts again.

The ability of controller (3.9) to decrease $k$ enables the system to stay sensitive. If $k$ has been increased to counteract a strong disturbance, the system becomes too stiff, should the disturbance subside or be replaced by a much smaller stimulus. When $k$ can be reduced, the sensitivity of the system grows and can therefore react to smaller disturbances. The existence of the reduction term ensures that $k$ is as high as necessary and as low as possible. The drawback of this design is that $y$ may leave the $\lambda$-tube repeatedly, since increase and decrease of $k$ are alternating.

### 3.4 Adaptors with simultaneous increase and decrease

So far, the adaptors have had some sort of scheduling for the different terms responsible for increase and decrease of $k$. In terms of minimizing the adaptor’s complexity, it would be advantageous to include all the desired behavioral aspects of the adaptor into one single equation. An attempt to do so can be found in [18]:

$$\textbf{Adaptor 6: }\dot{k}(t) = -\sigma k(t) + \gamma (\max\{0, ||y(t)|| - \lambda\})^2,$$

(3.10)

with $\lambda > 0, \sigma > 0, \gamma > 1$.

In literature, this is known as $\sigma$-modification (see [22], [12]). Note that this is a controller for $\lambda$-stabilization purposes only. To achieve tracking, $y(t)$ has to be replaced by $e(t)$.

The term $-\sigma k(t)$ decreases $k$ exponentially, while the second term ensures a quadratic increase of $k$ when $||y||$ is outside the $\lambda$-tube. Therefore, the two terms are active simultaneously and counteract
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Each other. Depending on the situation, one of the terms overcomes the effect of the other and results in a global decrease or increase of \( k \). This leads to chaotic behavior (see [21]).

Another adaption law that follows this philosophy, but avoids the drawback of the latter, can be found in [10]:

\[
\text{Adaptor 7: } \dot{k}(t) = -a \left(1 - b \text{sign} \left( \max \{0, \|e(t)\| - \lambda\} \right) \right) k(t),
\]

(3.11)

with \( \lambda > 0, a > 0, b > 1 \).

This adaptor distinguishes between the error value inside and outside the \( \lambda \)-tube. While \( \|e\| < \lambda \), \( k \) is exponentially decreased with \(-ak\). Otherwise, \( k \) is exponentially increased by \((-a + ab)k\). Note that there exists no simultaneous increase and decrease. Additionally, the parameters \( a \) and \( b \) have to be set by the user in advance.

4 Adaptive \( \lambda \)-tracking: a new first-order controller

Adaptive controllers with modifications of both adaptor and feedback law are considered.

4.1 Variation of the reduction term

So far, the term responsible for reducing the gain factor \( k \) has been set to an exponential decrease with a constant parameter \( \sigma \), see controller (3.9). A fixed linear decrease of \( k \) might be too slow, if \( k \) is large. An exponential decrease ensures faster reduction at high levels of \( k \) and slower reduction at low levels of \( k \). In Adaptor 5, the point in time when \( \|e\| \) enters the \( \lambda \)-tube is denoted by \( t_e \). The gain \( k \) decreases as soon as \( t - t_e \geq t_d \) and as long as the system output does not leave the neighborhood \( \lambda \). When that happens, \( k \) will be increased again. To avoid oscillation of increase and decrease, the reduction term is altered to be a non-negative valued function \( \delta(\|e\|, \lambda) \) of the error \( \|e\| \) and \( \lambda \):

\[
\text{Adaptor 8: } k(t) = \begin{cases} 
\gamma (\|e(t)\| - \lambda)^2 \quad & \|e(t)\| \leq \lambda + 1 \\
\gamma (\|e(t)\| - \lambda) \quad & \|e(t)\| < \lambda + 1 \\
0 & \|e(t)\| < \lambda \land t - t_e < t_d \\
-\delta(\|e(t)\|, \lambda) k(t) \quad & \|e(t)\| < \lambda \land t - t_e \geq t_d 
\end{cases}
\]

(4.1)

with \( \lambda > 0, \gamma > 1, t_d > 0 \).

The function \( \delta(\|e(t)\|, \lambda) \) is designed in such a way that it will decrease smoothly while \( \|e\| \) deviates from the set point trajectory and approaches \( \lambda \). Thus, the decrease of \( k \) is also diminished.

To achieve this, the function \( \delta(\|e(t)\|, \lambda) \) is, for example, set to:

\[
\delta(\|e(t)\|, \lambda) := \sigma \left(1 - \frac{\|e(t)\|}{\lambda}\right).
\]

(4.2)

with \( \lambda > 0, \sigma > 0 \).

Consider the case that the system is perfectly controlled, so that \( \|e\| = 0 \), and that the resting time \( t_d \) has passed. In this case, the decrease of \( k \) would begin with the full magnitude of \( \sigma \). When \( y \) moves away from the set point, \( \|e\| \) increases, which reduces the value of \( \sigma \) by the factor \( \left(1 - \frac{\|e\|}{\lambda}\right) \). As soon
as the system output reaches the border of the specified tube, $\|e\|$ approaches $\lambda$ and the reduction is stopped.

Figure 6 shows the simulation of the system with the old and the new reduction term. The parameters used here are: $\kappa = 0.1$, $\gamma = 150$, $t_d = 2$, $\sigma = 0.5$. To show the performance of the function

$\delta(\|e(t)\|, \lambda)$, the simulations are conducted without any excitation signal and with a set point trajectory of $y_{ref}(t) \equiv 1$. This requires the controller to keep a certain level of $k$, while it can be reduced as long as $y$ stays inside the $\lambda$-tube. The absence of any disturbance shows the abilities of the new reduction term much more clearly: The decrease of $k$ progresses much smoother, as shown in Fig. 6, right. It is no longer a strictly exponential decrease, but rather one restrained by the proximity to the outer borders of the $\lambda$-tube. The reduction is halted, when $y$ reaches the edge of the $\lambda$-tube, which means that $\|e\| = \lambda$. Note that there no longer exists an alternating behavior of decrease and increase of $k$, while $y$ exceeds $\lambda$ repeatedly as shown in Fig. 6, left. The same simulations are conducted again, now with the excitation signal as before. Figure 7 shows the results of the simulation. As expected,

the decrease of $k$ is smoother with the new reduction term. Also, the oscillation of $k$ is not as intense. However, the overall performance of the system is only slightly improved.
4.2 Tuning of $\lambda$-tube

Let $\lambda > 0$ be chosen in regard of certain requirements given by the context. One trivial way to ensure that the system output $y$ stays within that $\lambda$-tube is to use a smaller safety radius $\varepsilon \lambda < \lambda$ in the adaptation law (see [10]).

However, the appropriate value for $\varepsilon$ depends on the system and the excitation signal. In general, a smaller value for $\varepsilon$ leads to a higher value for $k$, which might result in a stiff system. This is to be avoided to keep sensitivity high. An $\varepsilon < 1$ improves the system performance only insignificantly. Simulations have shown that for the systems and excitation signals discussed above (with $\varepsilon = 1$), a value of $\varepsilon = 0.7$ is an appropriate compromise ([20]).

4.3 Results

Recapitulating all the investigations made in this section, there have been a number of findings that improve the classical adaptor:

- The introduction of the derivative gain parameter $\kappa$, which ensures stability through the existence of a D-gain. Simulations have shown that a small value of $\kappa = 0.1$ is sufficient for stability purposes and at the same time is small enough to not lengthen the settling time of the system.
- The improved decrease function for $k$, which lessens the reduction of $k$ in proximity to the borders of the $\varepsilon \lambda$-tube. Thus, $k$ does not oscillate as much while $y$ enters and leaves the $\varepsilon \lambda$-tube due to the to-be-recognized disturbance.
- A new adaptor that consists of only exponential gains for increase and decrease.
- The introduction of an $\varepsilon \lambda$-neighborhood in order to keep $y$ safely inside of $\lambda$ by referring to the smaller neighborhood $\varepsilon \lambda$ with $0 < \varepsilon \leq 1$.

Incorporating all findings into a new adaptor yields the following controller:

\[
\begin{align*}
e(t) & := y(t) - y_{ref}(t) \\
u(t) & = -k(t)e(t) - \kappa k(t)\dot{e}(t) \\
\dot{k}(t) & = \begin{cases} \\
\gamma (\|e(t)\| - \varepsilon \lambda)^2, & \|e(t)\| > \varepsilon \lambda + 1 \\
\gamma (\|e(t)\| - \varepsilon \lambda)^{0.5}, & \|e(t)\| \leq \varepsilon \lambda + 1 \\
0, & \|e(t)\| < \varepsilon \lambda < \varepsilon \lambda + 1 \wedge t - t_e < t_d \\
-\sigma \left(1 - \frac{\|e(t)\|}{\varepsilon \lambda}\right) k(t), & \|e(t)\| < \varepsilon \lambda < \varepsilon \lambda + 1 \wedge t - t_e \geq t_d \\
\end{cases} \\
k(0) & = 0 
\end{align*}
\]

with $\lambda > 0$, $\gamma > 1$, $\sigma > 0$, $t_d > 0$, $0 < \varepsilon \leq 1$, $\kappa > 0$ to be chosen in advance.

5 Adaptive $\lambda$-tracking: finite time behavior

In particular for practical reasons one could want the controller to have finite time capabilities. That means, the controller ensures a prescribed settling time $T$ such that the output $y$ enters the $\lambda$-tube no later than $T$ and never leaves the tube afterwards (if extra excitations are absent).

One finite time controller can be found in [16]:
\[ e(t) := y(t) - y_{ref}(t) \]
\[ u(t) = -k(t) e(t) \]
\[ k(t) = k_0 + \gamma_1 \int_0^t d_\lambda e(s) ||e(s)|| ds + \gamma_2 \left\{ \frac{||e(t)||^2}{T-t}, t \in [0,t^*], t \geq t^* \right\} \]
\[ d_\lambda (e) := \begin{cases} (||e(t)|| - \lambda), & ||e|| \geq \lambda \\ 0, & ||e|| < \lambda \end{cases} \]
\[ k^* := \frac{||e(t^*)||^2}{T-t^*} \]
\[ t^* := \min \{t \in [0,T); \|e(t)\| = \frac{3}{2} \lambda\} \]

with \( \lambda > 0, \gamma_1 > 1, \gamma_2 > 1, T > 0 \).

It shows several limitations and drawbacks. In order to overcome them, various modifications are made to the adaption and feedback laws in the following, due to [20].

**First limitation**

The controller (5.1) owns only as a proportional feedback controller due to a different system class in [16]. Since we need a PD-structure we add the term \(-\kappa k(t) \dot{e}(t)\) to the feedback structure.

\[ u(t) = -k(t) e(t) - \kappa k(t) \dot{e}(t) \]

**Second limitation**

In order to reach the finite time goal at \( t \leq T \), the gain \( k \) may be increased arbitrarily. This is the result of a pole that exists in the adaptor equation. However, such a sudden and strong increase of \( k \) causes the ODE system to become very stiff, which is problematic in numerical simulations. Also, a large value for \( k \) implies that the physical controller has to be able to generate an arbitrarily large control input, which is not feasible, and it causes an undesirable insensitivity of the sensor system.

**Third limitation**

There exist two control parameters in controller (5.1): \( \gamma_1 \) and \( \gamma_2 \). They have to be set by the user in advance and are therefore not part of the adaptive nature of the controller. It would be advantageous to modify the controller in such a way that the setting of \( \gamma_1 \) and \( \gamma_2 \) can be done independently of the system parameters.

In order to resolve these problems, various terms with finite time behavior are introduced into the controller equations. They also have a singularity at a specified time \( T \), so that \( k \) may be increased infinitely, if necessary. The goal is to find a function \( g_f(t,T) \) with a modest growth rate that supports the increase of \( k \) well before \( T \), so that no sudden increase resulting in a stiff ODE may occur. Some new finite time terms are:

- the addition of a linear term to the existing one: \( g_{f1}(t,T) := \left( \frac{T}{T-t} + \frac{T}{T-t^*} \right) \) with another parameter \( T \),
- the inverse hyperbolic tangent: \( g_{f2}(t,T) := \text{atanh} \left( \frac{t}{T} \right) \),
- a term using the natural logarithm: \( g_{f3}(t,T) := \ln \left( \frac{T}{T-t} \right) \).
These terms are put in the controller instead of the standard finite time term:

\[
k(t) = k_0 + \gamma_1 \int_0^t d_s(e(s)) \| e(s) \| ds + \gamma_2 \left\{ \frac{\| e(t) \|^2 \cdot g_f(t, T)}{k^*}, t \geq t^* \right\}
\]

with \( \gamma_1 > 1, \gamma_2 > 1, T > 0, k^* = \| e(t^*) \|^2 \cdot g_f(t^*, T) \).

To evaluate the performance of each modification, numerical simulations have been conducted in [20] to determine the minimum values for \( \gamma_1 \) and \( \gamma_2 \), at which the simulation is carried out without any numerical errors. If the parameters are lowered too far, the system becomes too stiff and the numerical simulation halts with an error message. This indicates that the setting of \( \gamma_1 \) and \( \gamma_2 \) would generate an unfeasible adaptive controller. Although this method can only provide an exemplary investigation, it allows a comparison between the finite time term modifications: The simulations show that if the finite time terms are turned off, i.e., \( \gamma_2 = 0 \), the value for \( \gamma_1 \) is the same for each modification. This is to be expected, as the adaption laws are equal without their finite time terms.

The linear term modification shows a nearly constant relation between \( \gamma_2 \) and \( \gamma_1 \) (see Fig. 8). This means that the setting of \( \gamma_1 \) is almost irrelevant, as long as \( \gamma_2 \) is set to a certain value. In this example, it is never greater than 40, which is also the lowest overall value for all terms investigated here. Therefore, the linear term is superior to the other modifications concerning the setting of \( \gamma_2 \) and \( \gamma_1 \).

The improved gain parameter is:
\[ k(t) = k_0 + \gamma_1 \int_0^t d_\lambda(e(s)) ||e(s)|| ds + \gamma_2 \left\{ \lambda^{0.5} \left( \frac{T}{T-t} + \frac{T}{T} \cdot t \right), t \in [0,t^*) \right\} \]

\[ d_\lambda(e) := \begin{cases} 
( ||e(t)|| - \lambda ), & ||e|| \geq \lambda \\
0, & ||e|| < \lambda 
\end{cases} 
\]

\[ t^* := \min \{ t \in [0,T); ||e(t)|| = \frac{3}{4} \lambda \} \]

\[ k^* := ||e(t^*)||^{0.5} \left( \frac{T}{T-t^*} + \frac{T}{T} \cdot t^* \right) \]

where we use an exponent of 0.5 due to observations made before.

**Fourth limitation**

There exists no term for decreasing \( k \) in controller (5.1) and (5.2). However, this feature is absolutely necessary.

In controller (4.3), the reduction of \( k \) is achieved by an exponential decrease term after a predefined time \( t_d \) has passed. It would be advantageous to include this capability into the finite time controller. However, this cannot be achieved easily. The adaption law for determining \( k(t) \) in (5.1) gives an explicit equation for the gain parameter \( k \), not for \( \dot{k} \). It is a zero-order controller, since there exists no ODE inside.

An exponential decrease term is much easier to implement into a first-order controller. Thus, the finite time capability is transferred from controller (5.2) into controller (4.3).

In order to do so, the derivative of the finite time term in controller (5.2) is computed:

\[ \dot{k}(t) = \gamma_2 \left\{ \frac{1}{2} ||e(t)||^{-0.5} \text{sign}(||e(t)||) \dot{e}(t) \left( \frac{T}{T-t} + \frac{T}{T} \cdot t \right) + ||e(t)||^{0.5} \left( \frac{T}{(T-t)^2} + \frac{T}{T} \right), t \in [0,t^*) \right\} \]

\[ 0, \quad t \geq t^* \]

The term is added to the increase terms for \( \dot{k} \) in controller (4.3), and the new controller incorporating all improvements now reads:
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\[ e(t) := y(t) - y_{\text{ref}}(t) \]
\[ u(t) = -k(t) e(t) - \kappa k(t) \dot{e}(t) \]

\[
k(t) = \begin{cases} 
\gamma \left( \|e(t)\| - \lambda \right)^2 + f_T(t), & \lambda + 1 \leq \|e(t)\| \land t < T \\
\gamma \left( \|e(t)\| - \lambda \right)^2, & \lambda + 1 \leq \|e(t)\| \land t \geq T \\
\gamma \left( \|e(t)\| - \lambda \right)^{0.5} + f_T(t), & \lambda \leq \|e(t)\| < \lambda + 1 \land t < T \\
\gamma \left( \|e(t)\| - \lambda \right)^{0.5}, & \lambda \leq \|e(t)\| < \lambda + 1 \land t \geq T \\
0, & \|e(t)\| < \lambda \land t - t_e < t_d \\
-\sigma \left( 1 - \frac{\|e(t)\|}{\lambda} \right) k(t), & \|e(t)\| < \lambda \land t - t_e \geq t_d 
\end{cases}
\]

with \( \lambda > 0, \sigma > 0, \gamma > 1, t_d > 0, T > 0, \overline{T} = 1, \kappa > 0; \lambda \) might be replaced by \( \varepsilon \lambda \) with \( 0 < \varepsilon < 1 \).

Using the parameters \( \kappa = 0.1, \varepsilon = 0.7, t_d = 2, \sigma = 0.5 \) and now \( \gamma = 10 \) and \( T = 5 \), Fig. 9 shows the system performance.

As shown there, \( k \) increases just before\( T = 5 \) and the controller achieves good \( \lambda \)-tracking. We point out that \( \gamma \) can be chosen very small (\( \gamma = 10 \)). This improves the adaptive nature of controller, since \( \gamma \) might be chosen independently of the system.

Fig. 9 Simulation using the new finite time controller (5.3).

Adaptive \( \lambda \)-tracking: PID-control

So far, we only discussed P- or PD-controllers together with the need to add a D-term to the feedback law for stability reasons. When designing classic controllers for LTI systems, the usual approach is to implement a PID-controller. It consists of a proportional, a derivative and an integral term, each with its specific gain parameter.
The idea behind incorporating an integral part into the control feedback is the reduction of steady-state errors (see [26]). In this instance we consider the case that $e > 0$ and $\dot{e}$ are small: the proportional and derivative gains have little effect on the system now and might therefore be unable to diminish $e$ further in order to reach the set point. With the help of an integral feedback term, this flaw can be overcome. This term increases, as long as $e > 0$, and therefore contributes to the control output $u$. The longer the steady-state error exists, the more the integral part increases. Therefore, steady-state errors are eliminated.

The drawback is that an integral term may lead to an overshoot of the system output (see [25]). To implement the PID-feedback structure into the adaptive controllers, only the feedback law is extended:

$$e(t) := y(t) - y_{\text{ref}}(t)$$

$$u(t) = -k(t) e(t) - \kappa k(t) \dot{e}(t) - \eta k(t) \int_0^t e(\tau) d\tau$$ (5.4)

with $\eta > 0$. There now exists the additional tuning parameter $\eta$ in the feedback law. It can be chosen by the designer to implement a relation between P- and the I-gain. A guidance of how to choose $\eta$ is presented in [20]. It is shown that a controller benefits from a small I-gain: $\eta$ is chosen as $\eta = 0.5$ for further considerations.

A $\lambda$-tracker with finite time behavior is controller (5.3) with the PID-feedback-structure (5.4) from above and with $\lambda > 0$, $\sigma > 0$, $\gamma > 1$, $t_d > 0$, $0 < \epsilon \leq 1$, $T > 0$, $T = 1$, $\kappa > 0$ and $\eta > 0$.

As discussed above, the integral term shows its advantages when trying to eliminate steady-state errors. The reference trajectory $y_{\text{ref}} \equiv 0$ (tracking of the rest position) does not cause a steady-state error, since it is an equilibrium of the system (for $x_0 \equiv 0$). To investigate the benefits of the I-gain, a different set point trajectory of $y_{\text{ref}} \equiv 1$ is chosen and the simulations are carried out once again. We choose: $\lambda = 0.2$, $\sigma = 0.2$, $\gamma = 10$, $t_d = 2$, $\epsilon = 0.7$, $T = 5$, $T = 1$, $\kappa = 0.1$. Figure 10 shows that the PID-controller performs better in eliminating steady-state errors when the I-gain is introduced.

The system output $y$ oscillates around the center of the $\epsilon \lambda$-tube (Fig. 10, right), rather than its border (Fig. 10, left). Also, the final value for $k$ is much smaller ($k \approx 70$) with the I-gain than without it ($k \approx 120$).
6 Conclusions & outlook

This work was motivated by the biological vibrissa sensor system. The vibrissa receptor cells are in a permanent state of adaption to filter the perception of tactile stimuli. This behavior has to be mimicked by the artificial sensor system. The sensor system was modeled as a spring-mass-damper system with relative degree two and the system parameters are supposed to be unknown. To achieve adaptive damping of unknown excitations, adaptive controllers were applied. Existing adaptive controllers from literature were improved with respect to performance, sensitivity and capabilities. Various modifications of existing controllers were made and new controller designs were discussed:

- a smaller $\varepsilon\lambda$-tube is introduced to prevent $y$ from leaving the $\lambda$-neighborhood,
- the term for decreasing the gain factor $k$ is made dependent on the distance of $y$ from the border of the $\varepsilon\lambda$-tube to restrain the reduction,
- controllers with finite time behavior are investigated and implemented,
- and the feedback structure of the controllers is extended.

The new adaptive controllers performed well when applied to the vibrissa sensor system. Finally, we present the effectiveness of the new controllers and adaptors compared to the ones presented in Section “State of the Art”. We noted that the controllers are not really sensitive to extraordinary impulses while ensuring good $\lambda$-tracking. We choose the following excitation:

$$t \mapsto x_0(t) = 3 \left(1 + 3e^{-0.5(t-20.5)^2}\right) \cos(t)$$

with an amplitude peak around $t = 20.5$.

First, we apply adaptive controller (3.4) with Adaptor 1. We obtain while varying $\gamma$ to ensure a better tracking: Figures 11 and 12 show that the system works best with $\gamma = 50$. In this case, the system is still clearly sensitive to the disturbance peak around $t = 20.5$, while $\lambda$-tracking is achieved during the rest of the simulation. If $\gamma$ is set to 150 or 500, the system becomes insensitive, since the disturbance peak is almost invisible in the system output as well as in the gain parameter $k$, which gives an almost flat line during the disturbance peak, as shown in Fig. 12.

If we use the existing Adaptors 6 and 7 we obtain the following: With Adaptors 6 and 7, the system stays sensitive to the peak in the disturbance signal while achieving a moderate $\lambda$-tracking, as
shown in Figs. 13 and 14. However, with Adaptor 6, the system repeatedly exits the $\lambda$-tube. The disturbance is clearly visible in both the system output (unsatisfactory tracking) and the gain parameter. Adaptor 7 shows better $\lambda$-tracking and the disturbance is also clearly detectable in the gain parameter. Controller (5.3) achieves the best $\lambda$-tracking results, since the $\lambda$-tube is never left. The transient response of the system at the beginning of the simulation (see Fig. 14, right) is not discussed, since it only depicts the initiation phase of the adaptor which is irrelevant for the control performance dur-
ing the rest of the simulation. However, the disturbance peak is hardly detectable, not at all in the system output and only very poorly in the gain factor. Therefore, sensitivity of this controller is low. The disturbance may be detectable when looking at the control input value $u(t)$. In order to keep the system output inside the $\lambda$-tube while under influence of the disturbance peak, the controller and the adaptor have to react with an increase in the control value. This should be clearly visible and therefore the sensitivity of the system can be ensured by considering the control input. This would solve the conflict of interest of generating good $\lambda$-tracking performance without neglecting sensitivity at the same time.

References