Lie, Santilli, and Nanotechnology:
From the elementary particles to the periodic table of the elements

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Abstract. Santilli’s revolutionary iso-, geno- and hypermathematics have provided the original straight line Lie groups and algebras with a span and coherence in all dimensions, and thus already at the infinitesimal level an extension in the Cartesian sense, allowing a continuous self-similar cyclical realization of matter from the elementary particle threshold level via the atomic to molecular and visible scale where it meets and marries with modern nanotechnology in the form of an isotropic vector matrix of space-filling octahedron-tetrahedron composition. This is distributed as an electron transition matrix with Bohr shell model stratified signature and is here directly outlining a new, centrally coordinated organic composition and chart of the periodic system as specifically exemplified by the noble gases.

1 INTRODUCTION

In this international year of crystallography, a coinciding important anniversary is that it is 125 years since Wilhelm Killing introduced his Wurzelsystem [1] in order to classify Marius Lie’s then recently disclosed [2,3] continuous transformation groups and algebras over the field of complex numbers. However, it has more and more emerged that the elegant vector diagrams that the root system comprises directly line up with real Euclidean space, too, where - and in that capacity also referred to as crystallographic - they quite by themselves lay out, combine and perpetuate the manifold lattice arrangement varieties found in Nature already at the infinitesimal level. Attesting this evidence of a coherent nanotechnological aspect and constitution of Nature and giving it canonical universal algorithm, the corresponding effective hybridization into ordinary structural laws and formulas has been realized by the Lie-admissible algebras of Ruggero Maria Santilli’s powerful iso-, geno-

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and hypermathematics. [4-6] Emulating their principal lifting of the trivial +1 unit into a bi-valued \(n\)-dimensional isotopic element compound incorporating the factors and operations of the equation into (iso-)unit value, allowed me many years ago in the simplest case of the \(A_2\) root vector diagram of SU(3) to duplicate it with its commutator (Fig. 1) and thus making “according to Santilli… a transition from contemporary physical models based on Lie algebras or their graded-supersymmetric extensions to the general Lie-admissible models, which transition essentially permits the treatment of particles as being extended and therefore admits additional contact, non-potential and non-Hamiltonian interactions”. [5]

![Figure 1](image)

**FIGURE 1.** When one \(A_2\) root space diagram (a) is inscribed in the unit sphere the vector network is plane (b), and (c) does not connect three-dimensionally, but when the commutator diagram is accommodated (d) a full Lie-admissible lattice results (e-g).

2 METHODS AND RESULTS

By that effective \(SO(3) \times O(5)\) automorphic coset decomposition of SU(3) over the real numbers, a hexagonal spaceframe lattice is generated and octagonally accommodated in classical three-dimensional Euclidean space (Fig. 1 d-g) that is the cubical framework (Fig. 2a) for Lie’s original geometriske transformationer “between the Plücker line geometry and a spatial geometry whose element is the sphere” by a “polygon… projection (of the Plücker cubical) space’s straight lines into the sphere’s rectilinear generatrices” [2,3] – which, in turn, are just the duplicated \(A_2\) diagram (Fig. 1d). [7-13] As previously reported, this essentially digital ‘eightfold eightfold way’ [10-14] system, congenial to all provenience and also progression - including Higg’s [13] - enables an exhaustive and exact reproduction of the parental ‘cuBit’ eigenvector field with instant representation and solutions of the Diophantine equations, Fermat’s last theorem and Beal’s conjecture [11-15] as well as of the elementary particle spectroscopy by volume- and spherical symmetry-preserving infinitesimal “curve-net”[2,3] transformations and projections between straight and round therein [7-14]; verbatim as the true Lie prescriptions direct. [2,3] This has been amply accounted for up to the baryons and mesons but not yet fully in the lepton [9-13], so the aim of the present paper is to continue with these and in especial the electron and subsequent atomic formations, producing, partially by the principles of octagon-square periodic tiling a new, organic composition and chart of the periodic system as here specifically exemplified by the noble gases.
2.1 An Isometric Vector Matrix Realization of the Electron

Fig. 2 illustrates the outward continuation of the duplicated root vector diagram between the unit sides, in this case the upper left of the eight Cartesian segment ‘cuBits’ of a Lie algebra neighborhood of Euclidean space (Fig. 2a) “into the sphere’s rectilinear generatrices” [2,3] (= infinitesimal generators) in one of the eight corners flanking the central spherical Nucleon domain (Fig. 2b,c). Further outward iteration sets up a lattice which in the 60 - 120° connections next dealt with (Fig. 3 a,b) consists of two half-octahedron sides of charged root vectors alone (Fig. 2b,c). In the case of starting from the upper left Cartesian segment cubit in Fig. 2a one can visualize what happens as that its four vertical sides are deflected 60° from the sphere’s nearest corner vertex, or tangential “image point” [2,3], creating the top of a 2\( \times \)2 high octahedron whose sequence continues upwards in a parallelepiped column (Fig. 2b) but may also

![Fig. 2](image1)

**FIGURE 2.** a) The generatrix [2,3] of Euclidean space is the Lie algebra neighborhood of the Plücker straight line geometry outlined by the eight infinitesimal Cartesian segments in it. b) In any of the two times four Cartesian quadrants from and around the spherical nucleon center, here the upper left, the diagonal (“quark-skewed”) root vectors span a lattice branching outwards as sections of a space frame isotropic vector matrix (c), be horizontally connected, adding two tetrahedrons to an octahedron interval (Fig. 3a) into a combined space-filling ‘singlet’ coil construction element of twelve unit sides, just as in the cube and like this a regular solid embodiment. (Fig. 3b) However, it is not quite as straightforward as that, because all parent cuBits share all sides with the flanking ones so that in the total transition also the horizontal sides are mobilized in a systematic way, step by step dragging in the neighbors. As shown in the montage below (Fig. 3a), when the obtained sections (Fig. 2b,c) are three-dimensionally tessellated (here just over the x and y axes), the resulting lattice is a stacking of the well-known, only other than the cube space-filling regular solid constitution, namely, a complex of one octahedron and two tetrahedrons. (Fig. 3a,b)

**FIGURE 3.** (a) A collage of the two-fold half-octagon sections, forming between them an outwards connectable lattice of one-octahedron/two tetrahedrons constitution, which (b) is the only other than the cube space-filling regular solid unit convolution, consisting of equally many, but deflected sides – the horizontal lines are Cartesian coordinate references. In (c), two open-ended varieties of the 12-side “singlet” electron coil are shown, that can continue into the full module.

It consists like the cube of twelve, however, all diagonally ‘quark-deflected’ [10-14] unit sides (the horizontal lines in Fig. 3b are the Cartesian coordinate references). The sideways periodicity, that is, translational symmetry and continuation of the singlet blocks is seen in Fig. 4a where the next
block is the Lie-admissible commutator mirror-image of the preceding, and thus taking part in forming a both horizontally and vertically periodic lattice. This is an essential nanotechnology structure, whose modular building elements, sections and assembly can thus be realized as a concentrically interlacing phase transition [7-13] at the very threshold of material arrangement, i.e., an ordinary space frame (Fig. 4b), alias octahedron-tetrahedron or octet truss, whose simultaneous space occupation implies an instant isotropically distributed universal inflation. As seen in Fig. 3a, the local space frame sections propel from a Cartesian segment of the Lie neighborhood, whose spherical root space thus abides as the hub, which is most pertaining in relation to the octet truss cast because, “viewed mathematically, as a generic pattern, its hubs correspond to the centers of spheres of equal radius”. [16]

In uniform lattice distribution this is what a Buckminster Fuller called “an isotropic vector matrix (IVM). . . with everywhere-the-same energy conditions” and “emphasized over the more familiar xyz apparatus arguing that our fixation on cubes and right angles goes against the grain of nature”. [Ib.]

**FIGURE 4.** a) Continuation of the root vectors outside the nucleon aligns as a sequence of the space-filling one octahedron/two tetrahedrons regular solid complex with twelve unit sides and translationally as well as vertically periodic and (b) which in iteration goes into a so called octahedron-tetrahedron truss space frame more technically referred to as isotropic vector matrix inclined to hierarchical modular expansion by its central co-ordination.
Once having an isotropic vector matrix, there are many options with important technical, architectural, physical, chemical etc. applications and expansions for space-filling agglomerations, smoothly and without holes fitting in with each other and on the whole. Here, for matters of demonstration, the singlet coils were first mapped into half a space-filling truncated octahedron configuration (Fig. 5). It is a space-filling Archimedean solid identified by Kepler to provide the optimal packing of spheres in space, and in the continuous plotting here of its frontal half-plane (Fig. 5a) is seen to come back to its IMV module hub 180° twisted and one step translated so that in setting out for the next, oppositely rotated round adjacent to the former also its hub is one step each translated over its directly touching, proton root vector axis and over one or two – building up a neutron excess – of its embedding cubical Euclidean space axes. That is, for the full flight of steps of the module balances one step of the central hub root vectors, which allows a calculation of the relative inertia. In Fig. 5b the sheet-wise distribution of the twelve-sided ‘octetrahedral’ units of the module formation bears out a correspondence with the so far observed Bohr orbital disposition, giving a total sequence length of \(152 \times 12 = 1824\) steps. The mass number in the leptons’ anti-symmetric \(U(1)\), inverse relation to that of the proton is therefore \(938.27/1824\) MeV = 0.514 MeV compared with 0.511 MeV of the electron. [11-13]

\[
\begin{array}{cccccccccc}
K & 2 \\
L & 8 \\
M & 18 \\
N & 32 \\
O & 32 \\
P & 32 \\
Q & 18 \\
R & 8 \\
S & 2 \\
\hline
152
\end{array}
\]

\[\text{Mass} = \frac{938.28}{12 \times 152} = 0.514 \text{ MeV}\]

**FIGURE 5.** (a) The two open-ended varieties of the singlet electron coil (Fig. 3c) can join to a non-rotational half-turn continuous delineation of an IVM module, (b) here portrayed as the truncated octahedron, according to a palindromic Bohr shell model distribution with the singlet coils in each layer represented as rods.

However, when investigating elementary particle generation as a polygonal surface crystallization process, the inherent alternative to the truncated octahedron’s outlining of the electron module in \(12 \times 152\) steps is offered by the electron singlet coils both vertically and laterally forming a coherent mirror-duplicated train with parallelepiped envelope (Fig. 4a, 6a) which, following the same to and fro *Zitterbewegung* sequence as in the truncated octahedron (Fig. 5b) packs a trapezoid stack of \(2+8+18+32+32+32+18+8+2 = 152\) unit bricks. (Fig. 6b)

**FIGURE 6.** a) 12-step spaceframe lattice singlet coils are mirror-distributed in both vertically and laterally coherent parallelepiped envelopes (here shown as cubes) b) which may be tied together, in
the same Zitterbewegung $2+8+18+32+32+32+18+8+2 = 152$ unit brick coherent sequence of palindromic Bohr orbital distribution as in the truncated octahedron, into a trapezoid stack module able to hierarchically self-expand around a spherical IVM hub.

Fig. 7 gives an overview of this direct nanotechnological construction module and its self-assembly modes, akin to “the self-organizing specialized bricks” recently described in “robot manipulation of a shared environment... built from square bricks in a non-overlapping grid pattern”. [17]

**FIGURE 7.** (a) The 12-step singlet electron coils are here pieced together as infinitesimal ‘cuBits’ in a palindromic Bohr orbital model distribution. Actually, they are of parallelepiped span (Fig.s 4a, 6a) but that doesn’t change their continuous linking according to Fig. 5a by a non-rotating Zitterbewegung IVM outline (Fig.s 5b, 6b) into a 1824-step half-turn module with proportionally higher inertia = lower mass in relation to the central nucleon hub, and then ready to proceed the modular course (b-e), interchangeably either side-ways (b-d), upwards (e) or mirror-wise (f-h). It is seen that the assembly leaves a central vertically (c,d) and horizontally (e) connected groove for the convoying IVM center. In the mirror IVM compounds reciprocal cavities appear (f-g).

The idea that electrons may be both wave functions, here the cloverleaf singlet coils, and “transition matrix elements”[18], here the IVM modules, is not new but in the present cast embodied to a
coherent lattice that corresponds to the fact and way that they tangibly weave the world. Like everything else on a digital basis they are but evading unit line bits when pursued to their origin but from there they are caught in the global phase transition “between the Plücker line geometry and a spatial geometry whose element is the sphere” by a “polygon…projection of space’s straight lines into the sphere’s rectilinear generatrices”[2,3] that Lie’s original geometriske transformationer in detail define.

The other durable algorithm is the Bohr stratified apportioning of the building blocks whether conceived as points or bodies, and that this pattern proceeds from the elementary particle stage to and beyond the thousandfold larger atoms. The mechanism is self-similarity, when the modules both template and fill out a successive cyclical expansion of their motif (Fig. 8a) so that the atom cross-section size is reached within a limited number of cycles.

In the kick-off primordial Universe the serpentine route to the Hydrogen atom that together with some Helium prevails there reminds of the “3D island growth…pathway of layered growth” recently described at “angstrom-scale [and even single-atom scale] resolution” [19,20] in surface crystallization processes, where “the growth units are individual molecules and ions that attach

**FIGURE 8.** Equatorial plane cross-section of first three cycles of self-templating Hydrogen ion expansion (a) and the dismantling extrusion squeeze exerted by condensation impact in the nucleogenetic era (b) in monomolecular steps present on a crystal surface” [Ib.]. That the same modus operandi with identical topographical “step heights, terrace widths, and other surface features” [Ib.] comes up in several cycles from the very ground is then entirely consequential, and the objection that such intricacy cannot happen at such diminutive size can be met by comparing with the myriad times smaller and more complex strings. Quite simply; There is the bottomline unit digit. And it takes so and so many convolutions to shape us ground-up.

But that could not happen in the primordial universe, patches of which still persists, where the island modules peacefully wind up their isotropically dispersed space quota - and nothing more. Inevitably, however, there will be small fluctuations, approaches, collisions, condensations, perturbations, and soon enough the nucleogenetic era ensues, for which there is an explanation, too, in the present model. (Fig. 8b) This is that when a number of hydrogen ions are compressed against each other at sufficient energy to fuse, they will be squeezed side by side along a line perpendicular to the force so that from whatever transverse direction will result an axial duplication (Fig. 9a) like in the ends of the mirrored module compounds (Fig. 7f,g) with Bohr period 2+2, 8+8, 18+18, 32+32 etc.

**FIGURE 9.** a) Mirror juxtaposition of two electron modules with doubled Bohr periodicity. b) By the nucleogenetic pressure the module is flattened in its ends, packing the 152 singlets in a square brick with two open corners.
Due to the pressure the hydrogen ions are forced to uncoil until the last singlet is reached in each, meaning that they come together in the terminal module, where each occupies one singlet in a honeycomb arrangement where the number of cells in duplicated Bohr periodicity depends on how many hydrogen ions were engaged in the fusion.

2.2 Periodic Tiling of the Periodic System

Much of this is of course inferential but we know that there was a primordial Universe of largely Hydrogen, still persisting at places, and we know that there from fully understandably condensation events occurred a still ongoing nucleogenetic era, when the electron IVM fabric underwent the crunch of extreme compression with effects both upon the elements and their arrangement; in which, tentatively, a converging cloud of them is collapsing together in a narrowing module funnel (Fig. 8b) and there cyclically uncoiling under shedding off of photons and neutrinos and other fragments until in the ultimate end pairwise sandwiching up along the sides as a wafer of together equally many singlets as the cloud’s amount. Maybe not so crystal-clear formulated - but clear for re-crystallization by the formation of so individualized Hydrogen ion seeds by cross-sectional extrusion in an alternating sequence from the here vertically duplicated Bohr-periodic singlet “terraces”. [19,20] (Fig. 9a)

To continue the inferences, when the expansion resumes within these cells, there is still a high pressure causing in the first round a correspondingly contracted electron module. (Fig. 9b) This will self-organize and assemble according to the “spiral growth model…of 2D layer growth” [Ib.] that is natural in the composition of the flat, one- or more-storied type of IVM space frame (Fig. 10a) and to which the so modified module is ideally inclined (Fig 10b,c) both in area and modular expansion by square-octagon periodic tiling [21,22] and “cube-oct” structure build [23], respectively; the latter recently reported from the suitably named Center for Atoms and Bits at MIT [Ib.]; and both verbatim originally prescribed by Marius Sophus Lie in his thesis “Over en classe geometriske transformationer”. [2,3]

FIGURE 10. a) stratified type of IVM spaceframe. b) Cross-section of same type with two corner beams shown. c) With that self-organized “cube-oct” composition the spiral variety of surface crystal growth can proceed by stacking of the bricks (here not same number as in the module cross-section) like a flight of stairs in consecutively one layer higher piles, and possible to turn also vertically with then horizontally convoying center.
That the atoms are conglomerates of equally many hydrogen ions as their atom number was proposed already by William Prout in 1815 [24] but was abandoned until the discovery of isotopes gave it a new boost. Recently, number theory has given further strong support [25], but ways and means of implementation have been hard to figure out. The present results would serve to suggest a solution consistent with known stages and events in the natural history of the elementary particles and in especial the electron-atom interrelations. The virtual extrusion mechanism can be empirically tested in terms of its output as will be done here in a periodic tabular form with special reference to the noble gases. (Fig. 11) Referring to Fig. 9a, the most

**FIGURE 11.** Periodic chart of the noble gases. Only one of the two initially transferred hydrogen ions is shown to indicate its Fermion status. The coloring is according to the exaggerated color of the respective gas in the discharge tube. It is implied that the noble gases are saturated and released when a delivery pulse has been completed and optimally distributed, and that a further condition of the release is that they tile, as indicated, too, in the chart. Also some constellations of lower atoms filling up a full cycle along the margin may be tilable in principle but not released because of their engagement in a not completed pulse. Marginal extrusion would be two singlets from one half of the terminal terrace, resulting in two hydrogen ions joining to a Hydrogen molecule (Fig. 11) which is then free to leave the stage. When at higher energy also two singlets have been forced together in the other half, the ensuing pulse would fuse to the Helium atom, likewise free to leave in any direction when these are no further singlets coming. When there are, the Helium motif would be deposited in the centre, and the remaining sending would line up peripheral to it. The sending is complete when half, i.e. eight, of the singlet positions in the next terrace (Fig. 9a) were filled and then transferred into the next thus saturated noble gas, i.e. Neon. And so it continues; Argon when the eight singlets in the remaining half of the terrace have been delivered; Krypton when the eighteen singlets in half of the next terrace have followed; Xenon when the eighteen in the remaining half have; and then there are 32 in half of the next terrace and when this number is added there is Radon. Then there should take 32 steps to the next state, but already at 28 occurs Flerovium - plausibly because at this high atom number it is still readily tiled (Fig. 11) - before indeed at 32 the heaviest noble gas, Ununoctium really appears at atom number 118.

Hence, the duplicated Bohr orbital model periodicity of the periodic table is generated by the cyclical delivery from a bilateral transfer structure which deposits Hydrogen ion seeds in a cross-sectional fiber pattern, then consecutively expanding within each cell by the inferred spiral “2D layer growth”. [19,20] Only the noble gases are shown here, but the other atoms and their properties are possible to fit into their respective boxes in a thus centrally coordinated, organically constituted periodic system, where also the states have counterparts in the number of cycles to modular completion; highest in the solid atoms, one cycle less in the fluid and two cycles less in the gaseous.
Hydrogen ion

Helium $K_2 = 2$

Neon $K_2, L_8 = 10$

Argon $K_2, L_8, M_8 = 18$

Krypton $K_2, L_8, M_{18}, N_8 = 36$

Xenon $K_2, L_8, M_{18}, N_{18}, O_8 = 54$

Radon $K_2, L_8, M_{18}, N_{32}, O_{18}, P_8 = 86$

Flerovium $K_2, L_8, M_{18}, N_{32}, O_{32}, P_{18}, Q_4 = 114$

Ununoctium $K_2, L_8, M_{18}, N_{32}, O_{32}, P_{18}, Q_8 = 118$

?-ium $K_2, L_8, M_{18}, N_{32}, O_{32}, P_{18}, Q_{10} = 120$
A periodic table of the present layout further overcomes the standard version’s deficit problems of the empty initial gaps between Beryllium and Boron and Magnesium and Aluminum as well as the surplus dilemma of the terminal appendixes of the Lanthanides and Actinides; lining up the elements organically and symmetrically along the margins of concentrically growing atom strata where apart from penultimate, such as halogen sites they may not differ much in chemical behavior. The exact morphology remains to be elucidated, however, since alternative to the compressed double-module mechanism (Fig 8a), the same hierarchical extrusion order can also be obtained in a successive inwards conical excavation sequence from the octet core of single modules. This mode is illustrated in a lateral projection in Fig. 12.

**FIGURE 12.** Lateral projection of noble gases in the reciprocal order to how the singlet cells and terraces (dotted are interior layers) are filled obliquely from the core so that, apart from in Helium, the sequence will always end in an octet trunk, thereby completing a turn and in consequence having no binding sites. The cross-sectional deposition of the Hydrogen ion seeds in the respective atoms will still be centrally patterned albeit tentatively in a more segmented arrangement.

### 3 DISCUSSION

Nanotechnology is rapidly coming down in scale and has now reached the atomic level, [19,20], all the way encountering hierarchically self-similar crystallographic lattice structures; usually, and in the inorganic realm almost exclusively of a regular solid composition identical to the present isotropic vector matrix generated via Marius Sophus Lie’s original continuous geometriske transformationer. [2,3] Thereby, Nanotechnology has carried to the very infinitesimal realm its rational Renaissance of un-dogmatic objective Natural History observation and operation whose deepest domain and idea can be summarized as extension: in the Aristotelian sense revived in Western science by Descartes and Lie alike. The latter even in his thesis stated that it “stands in an intimate dependence on philosophical reflections upon the nature of Cartesian Geometry” [2,3], where, in turn, the first principle is that “the extension in length, width, and depth which constitutes the space occupied by a body, is exactly
the same as that which constitutes the body... consequently, there cannot exist a space separate from
body, since all spatial extension simply is body”. [26]

The corollary is that space, which is extension, which is endless, consists of itself and is its own
eigenelement and therefore necessarily space-filling in and by itself, and that its alternative is not
emptiness or vacuum but nothing at all; the shrinking away and absence of any quality whatsoever,
against which the direct straight line eigenvector of extension stands out as the categorical antithesis;
the obligate obverse; equally immediate and raw and primary as its radical contrast. [11] And Nan-
otechnology does not deal with vanishing points but with concrete extended objects thus rising as
such from the outset. My contribution is to show that one can indeed, like the old Greek, by solely a
line and its maximally three-dimensional convolutions build a close facsimile of the real world with
both ancient regular solid Cosmology and modern IVM Nanotechnology signature, bridging from the
elementary particles to the elements – and onwards.

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